ITA Transcripts

Title:	Business Calculus 1
Focus:	Recitation
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Publisher:	TESOL
Context:	This transcript is from a calculus course that is required for business majors. The videotape was made in April 1999. The Turkish TA had experience teaching math before he came to the U.S. and he generally did an excellent job in engaging his classes, attempting to involve the class by asking questions and by providing real life examples. He also tended to have a very methodical way of approaching problem solving. Though the students in this particular class period were not as responsive as in another that I observed, one can notice the effort that the TA is putting into trying to involve them.

T: I want to take a look at section 7.1 and hopefully a little bit to section 7.2 . which is about ah functions of ah . two variables and eh I think this is kind of important subject for the exam [2 sec]eh this time eh things getting a little bit eh harder because we are goin going to be looking at ah (writes on board) not only one variable but two variables . so eventually we will look at some properties of two va ah functions of two variables in x and y .
first of all ah let me ask you this (2 sec) when do you use ah ah the functions of two variables . it is ah used for (3 seconds) I mean

1:00
ah let's le'ts think about science okay what if you have kind of (writes on board 7 seconds) function q of k and l . let' s define some function . lets say (writes on board) K cubed . L to the one/half . In that case K is capital (unintelligible) L is the ah size of labor that is labor force . measured in worker hours . also let's cite an example in economics [writes on board- 3 sec] and ah (writes on board-9 seconds) let's give an example in psychology [4 sec] let's define (writes on board) a function of two variables right, my first variable is m

2:00
the second one is a [2 sec] let's define this function 100 times m divided by a [2 seconds] where m is the ah person's mental age [4 sec writing on board] a is the person's actual age [writes on board-3 sec] right? and this is called IQ test [writes on board-3 sec] if you know the person's mental age and actual age you can compute the ah . intelligence quotient [2 sec] this is a function in two variables . okay . and there are of course many examples like air pollution and some (unintelligible) whatever. first of all ah ah after giving this ah examples [2 sec] I would like to talk

3:00
about the domain of f in two variables . remember? for the ah [2 sec] in one one variable case right it was kind of easy easier to find the domain [1 sec] but this time you have two variables . one is x and y [1 sec] and ah the first thing is ah we are supposed to find the domain of f . as we did before right? before drawing the graphs . what was the first thing we ah considered? [4 sec. Waits for students to answer question- no response] okay now

eh I will look at some examples and try to find out the domains of functions in two variables [4 sec] first of all ah let's remember ah the definition of domain .

4:00
what was the definition of domain? [2 sec] first of all it was the set of all pairs x and y such that you can compute the ah . value f of xy okay? [writes on board 3 sec] if want to define the set ah this domain [writes on board-3 sec] its [2 sec] it consists of all pairs such that [3 sec-writes on board] f of xy makes sense [writes on board 3 sec] okay [paces and looks at notes-9 sec] now lets look at some examples . does this definition make sense now? [2 sec] Just try to make this definition simple to the one dimensional case

5:00
okay [2 sec] now le'ts see eh some examples (writes on board) what if you have f is defined as [3 sec] x divided by ln of y . and we are supposed to find out the domain of this function remem uh see this function is . there are two variables in this function . x and y [3 sec] okay now my question is for which x and y does this function make sense? [waits for students to answer-8 sec] remember? ah we don't want the denominator zero right? we don't want this [2 sec]

6:00
so [2 sec] for which y the denominator is zero?
S: one
T: one very good [3 sec]
okay so one is dangerous point for this function . okay so we will exclude one from the eh domain . for sure [3 sec] what was the eh . domain for l and y? for which y . l and y makes sense?
S: 3 bigger than zero
T: y is 3 bigger than zero . very good . so y [2 sec] should be positive but ah you just said that y cannot be one . right ? but one is also positive 23 sec] right? so you should ah you should say that all ys but except one . all y's are positive

7:00
except one so you can [writes on board] is this clear everyone? [5 sec] just remember the ah graph of l and y okay? it was something like this [2 sec] right it was defined for all positive x not negative x [2 sec] okay and here is the point one okay so the domain of f . is consist of all pairs x and y . such that [2 sec-writes on board] y is strictly positive . comma [writes on board] y is not equal to one . okay all positive numbers except one [3 sec] and also note that there is no restriction

8:00
on x . because x the numerator is okay . because it's just a first order polynomial [4 sec. Walks across front of room looking at board then looks at student]
S: [unintelligible question]
T: excuse me?
S: [unitelligible]
T: [TA leans toward student to hear her] why? because at point y . l and y is zero right? [2 sec] remember? [2 sec. Taps diagram on board] l and one is zero do you remember this? I gave you last week I guess [2 sec] and its also clear from the graph . okay so if this zero then this function doesn't make doesn't make sense so you should exclude y equals one. [6 sec. Begins to erase board]
okay any other question on this one? [looks at students 4 sec.]

9:00

[erases board] okay now let's do another example . quick example [5 sec. Writes on board] what if you have f of xy is defined as . y squared minus x squared this time [4 sec] now let's find out the domain for this function [4 sec] remember I told you last time I guess . in the final exam there will be short questions . no credit or full credit [2 sec] so. maybe in the short answers . you know there will be some kind of questions . this kind of questions right? if you'll be given it just ask the domain if the domain

10:00
is not completely correct then there will be no credit . so try to understand here clearly the domain okay . now let's try to find out the domain of f for this function [5 sec. Waiting for students to answer] any suggestion?
S: (unintelligible)
T: let let's say why do you think its this thing
s: (unintelligible)
T: let's say y equals x [2 sec. Writes on board] okay? let's say for a moment if y equals x then this ah [2 sec] ah we have like the square root of zero which makes sense . right? the square root of zero is zero [3 sec] no dangerous point it eh y equals x [2 sec] what was the restriction of square root function

11:00
what was the domain of square root of x it let's say [2 sec] it must be positive . but it could be zero right? [2 sec] zero is perfect okay so . eh we want really y squared minus x squared to be non negative it can't be negative but could be zero [4 sec] now this implies that y squared should be bigger than or equal to x squared right? [2 sec] now from here what can I say? what's the relation between y and x . [7 sec] what if I take the square root of both sides [writes on board] do I only get this ? [3 sec] or there are more
S: [unintelligible] [TA walks down the center aisle closer to the student as he listens]
T: sure . very good. which means what . the absolute value of y . bigger than or both absolute value of x because y or x could be negative also right? [2 sec] so I should put absolute value on both sides [2 sec] because y or x could be negative right? [2 sec]

12:00
just remember this definition [writes on board] if someone ask you that . what what is this variable equal to . what is the square root of x squared [2 sec] and your answer should be absolute value of x right [2 sec] x could be negative. you can cannot just say this equal to x [2 sec] right? okay be careful about the sign positive negative always consider about these two things . could be zero could be negative could be positive [3 sec] okay just remember this rule . So the answer to this question is the domain is equal to . all pairs

13:00
x and y such that the absolute value of y greater than (unintelligible) absolute value of x [3 sec] okay? [3 sec] all right I think that's enough about the domain . functions? [starts to erase board] can I erase here [erases more] okay now eh [2 sec] let's ask this question . how do we graph . functions of two variables right eh this time things a little bit complicated because . you have two variables

14:00
to consider . and you should introduce three dimensional coordinate system . right in this case [writes on board] x y and z coordinate and you have a function [2 sec] lets say f of xy to plot . so you need another coordinate right? to plot the two dimensional variable [2 sec. writes on board] okay now lets introduce [2 sec writes on board] coordinate z and lets say z equals f of xy and its usually not easy . eh to graph this kind of functions without computer . it's almost eh impossible to draw eh .

15:00
two variables eh the functions of two variables in three dimensional coordinate system . but computers does it for you perfectly . but instead we are going to look at the level curves [2 sec] okay now let's say you have a point here x and y .[draws on board] okay and somewhere here . you introduce this number right z okay now this point [2 sec] is actually x y and z but what's my z? [2 sec] z is x of fy okay? but now you can draw a surface around this [3 sec] right which we will call level curve in a minute [3 sec. Goes to desk and looks at notes]

16:00
and eh to find the level curves [3 sec writes on board] level curves [2 sec writes] we will look at [4 sec. Writes] f of xy equals some constant c . which is a real number . okay? . for some [2 sec. Writes] various values [2 sec writes] of c [2 sec writes] and we will call [2 sec] this curves . to be level eh curves actually as I told you ah

17:00
a few minutes ago this is almost ah . impossible to draw the ah surface in three dimensions . but instead you we will look at two level curves because curves can be handled [2 sec] in two ah in two dimensions which we ah used to deal with . because its kind of y equals f of x which we know how to draw right? [2 sec] level curves . but surfaces are not easy to draw in three dimensions [4 sec. Look at notes] okay let's look at some examples [4 sec. erases board] by the way

18:00
before we continue I want to I want everyone to take a look at page eh five eleven and five twelve please? [5 sec Waits for students to find page.] at the bottom of page one five eleven . you will see . a surface which is red do you see that? and it's kind of difficult to draw it in three dimensions . but instead . it's easy to ah find out the level curves . here is the level curve is z equals c [2 sec] and this is the plane all you have to do is find the intersection between the eh plane z equals c and the surface . and also on page five twelve

19:00
in the first picture [2 sec] you will ah you will see the ah level curves right? for different values of c . first of all z equals hundred I mean c equals hundred . in the second level c equals three hundred . thousand and fifteen hundred . do you see that? [3 sec] and if you after finding all level curves if you put them together . okay? suppose you have found all the level curves and then you put them together and you get the surface back . okay think it this way [2 sec Looks around classroom] right? suppose you have let's say a potato. right ? and you slice it and each slice it gives you level curves but if you put them together back than you get the surface back . in three d

20:00
[4 sec] does it makes sense to you? [2 sec] okay [2 sec] now let's look lets look a the some level curves of functions [2 sec] number seven [2 sec writes on board] we have f of xy equals [2 sec writes on board] x plus two y . okay and [2 sec] we want to find the level curves for c equals one .

21:00
c equals two and c equals three because these are easy to draw in two dimensions [2 sec] let's say f [2 sec writes on board] c is one . what if c is one I will introduce f of xy is c right? as I did here and my f of xy is x plus two y equals one [2 sec] do I know how to graph this function? . of course I know right? because this should be very easy for us by now because we draw a lot about these kind of functions . if eh you solve this equation for y . y equals one half times one minus x and this is easy to draw in two dimensions x [2 sec draws] y . what if x equals one . y equals zero right [2 sec] here is the point one zero if x is zero then y equals [1 sec] one half

22:00
somewhere here [2 sec Draws] so I have a straight line right [1 sec] this is my? c equals one level curves okay? now similarly of course you can draw c equals two level curves . c equals three level curves. oh by the way this should be minus three . and there are kind of . [drawing] this is c equals two . and this is c equals minus three level curves and if you

introduce more c's . like c equals zero minus two minus five kind of things . and you get the surface back . right? [4 sec looks at notes]

23:00
all right [erases part of board] is this clear? [4 sec points at graph] okay . [writes] what if you have function y f of x y is given y y times e to the x and [2 sec] we are supposed to find out the level curves for c equals zero and c equals one . all right? [5 sec] now if c is zero . then I have y times e to the x is zero right?

24:00
[3 sec] now the question is . for which x and y . this equality holds [7 sec waits for students to answer. Looks at class.] you have a product of two things right? [5 sec]
S: [unintelligible]
T: hm?
S: [unintelligible]
T: excuse me?
S: [unintelligible]
T: which one is zero
S: [unintelligible]
T: y is zero this one implies that y is zero . what about the other part right ? if you have if the product of two things is zero . what can you conclude from that?

25:00
one of them should be zero right? . but can e to the x be zero for any x?
S: [unintelligible]
T: hmm excuse me?
S: [unintelligible]
T: no because e to the x is strictly positive . okay? it cannot be it can't even be zero for any x . if e to the x is strictly positive then there is only one possibility from here y should be zero . right? [2 sec] is this clear? okay so this is my . c equals zero level curve okay lets draw this

26:00
[erases board and writes new formula] so here is x here is y . how do you plot . y equals zero in two dimensions [8 sec Looks at class.]
S: [unintelligible]
T: pardon me
S: [unintelligible]
T: y axis? or x axis? x axis right okay . so all this axis all the ys are zero righ?t on this axis . so this my c equals [sec. Writes on board] zero level curve . now let' s find out what I what's [3 sec. writes on board] my c equal one level curve

27:00
if y times e to the x is one [4 sec. Writes on board] what's y? [9 sec. Waits for student to answer]
S: [unintelligible]
T: excuse me which is? e to the minus x . right? one over e to the ex is equal to? . e to the minus x . is this clear? because one over e equals e to the minus x . but I know how to plot this function [2 sec.] does anybody remember . how to graph this function . how is the graph looks like? . e to the minus x
S: [unintelligible]
T: excuse me?
S: [unintelligible]
louder please

S: [unintelligible]

28:00
T: very good . so which means like we are coming this way right hitting this point and going to zero. this is minus c equals one level curve [2sec] okay now for the last thing today I would like to show you some pictures that eh I computed on in the computer today for you [looks through the transparencies on the desk and places on one the overhead] let's look at couple of examples? [pulls down the screen and plugs in overhead projector]

29:00
oops oh I'm sorry [positions the overhead projector] okay [arranges transparency] is the picture clear? can every one see that? okay . now eh . this is the graph of z equals cosign x times cosign y times e to the minus one-fourth times the square root of x squared plus y squared in three dimensions . here is my x coordinate? here is y coordinate? and here is z coordinate and its almost impossible to draw it by hand

30:00
in three dimensions . right? but instead you can look at the level curves . and here I'm not sure if you can see that this curve here? this is the level curve at I think z equals zero point one if I'm not mistaken . and it's kind of a slice . in two dimensions [changes the transparencies] and here is another one [arranges the transparency] here is the [unintelligible] sphere in three dimensions . here is x coordinate y coordinate and z coordinate [3 sec.]

31:00
which is x squared plus y squared plus z squared is equal to one . but you can think you see these lines all around? and you can think these lines eh to be your eh level curves . at eh different c's . and then it will be easy to understand what is going on . because if you take any slice . right? if you take this slice . you will get a circle and you know how to draw the circles in two dimensions right? it's very easy [4 sec] okay I think that's all I want to say for today we will have the quiz in a minute

32:00